Aims & Objectives: The purpose of this course is to provide a compact coverage of special relativity using the 4-vector formalism with an introduction to tensors at the level needed for special relativity. The recommended reference book is *Introduction to Special Relativity* by W. Rindler (Clarendon Press; 1st ed, 1982, or 2nd ed, 1991; paperback).

Knowledge of special relativity equivalent to the first two of Rindler’s chapters will be assumed, but this will be supported by an initial revision lecture surveying the coverage of special relativity in First-Year Physics. Some references to parts of Rindler’s book are included below, but some sections of the course do not follow Rindler’s treatment. Relativistic electrodynamics is not covered, as this is dealt with elsewhere in Third- and Fourth-Year Physics.

Successful completion of this course will open up to the student a big range of physics literature involving special relativity, in fields such as particle physics and astrophysics. The course will prepare students for later study of general relativity, relativistic quantum mechanics, and quantum field theory. There will be a strong focus on the development of problem solving skills.

Assessment: This will be by two assignments worth 15% each and an examination worth 70%.

Outcomes: Expected outcomes include

- reinforcement of the knowledge of special relativity gained in First-Year Physics.
- an introductory knowledge of tensor methods
- well-developed problem solving skills in special relativity
- the background knowledge needed for studies of topics such as general relativity, relativistic astrophysics, and quantum field theory

Syllabus

Relativity Revision (1 lecture): A summary of special relativity at the level of First-Year Physics (Rindler: Chaps 1 & 2)

Spacetime: Lorentz transformations; Minkowski space; 4-vectors; 4-tensors; space-time “rotations”; classification of intervals; what is a tensor? (Rindler: Sections 20-22)

Kinematics: proper time; 4-velocity & 4-acceleration; velocity transformation; ‘superluminal’ motions in quasars (Rindler: Sections 23 & 13)

Dynamics: 4-momentum; energy-momentum relation; 4-force; 3-force; transformation of 3-force (Rindler: Sections 26-28, 35)

Optics: aberration of light; visual appearance; ‘global aberration’ (Rindler: Section 18)

Tensors: coordinate transformations; contravariant & covariant vectors; contravariance versus covariance; outer product & second-rank tensors; some special tensors; contraction; symmetry & skew symmetry; etc (Rindler: Appendix; and see separate notes)
Special Relativity Problem Sheet 0: Trivial Revision Problems

**TR1.** The nearest star system to the Sun, α Centauri, is 4.3 light years from us. A spaceship travelling at a constant speed gets there in 4.3 years as measured by the astronauts on board.
   (a) What is the spaceship’s speed, as a percentage of \( c \), according to observers on Earth?
   (b) How long has the journey taken according to observers on Earth?
   (c) How far do the astronauts on board consider that they have travelled?

**TR2.** After a spaceship has travelled at constant speed from Earth to a nearby star system, the astronauts on board consider that they have travelled 3.05 light-years. The journey has taken 6.1 yr according to observers on Earth.
   (a) How far is the star system from the Solar System, in the rest frame of those two systems?
   (b) How long has the journey taken according to the astronauts on board?

**TR3.** Evaluate the binding energy of a helium nucleus – *ie* the energy required to break it into its constituents, two protons and two neutrons. The rest masses of a proton, a neutron and a helium nucleus are 1.00783 u, 1.00867 u, and 4.00260 u, respectively. One u denotes one unified atomic mass unit, defined to be precisely one-twelfth of the mass of a neutral \(^{12}\text{C}_6\) atom; 1 u is equal to 1.6605 \(\times\) 10\(^{-27}\) kg, which is equivalent to 931.50 MeV of energy.

**TR4.** (a) In a special relativity problem, a student finds that \( \gamma v/c \) (with \( \gamma \) denoting the Lorentz factor corresponding to speed \( v \)) is a certain number \( N \).
   (i) How does the student work out \( v/c \) and \( \gamma \) separately?
   (ii) What are \( v/c \) and \( \gamma \) for \( N = 0.1, 1, 10 \)?
   (b) Show that \( \gamma \approx 2^{1/2}(1 – v/c)^{-1/2} \) when \( v \) is close to \( c \).

**TR5.** (a) What is the speed, as a fraction of \( c \), of an electron whose mass is 14,000 times its rest mass? (Such speeds are reached at the Stanford Linear Accelerator Center, SLAC.)
   (b) If the electrons travel through a tube 3.0 km long (as at SLAC), how long is this tube in an electron’s rest frame? (That is: How long is the tube as determined by a hypothetical observer moving along with an electron?)

**TR6.** If the kinetic energy \( K \) and the magnitude \( p \) of the momentum of a relativistic particle can be measured, then it is possible to find its rest mass \( m_0 \) and thus identify the particle.
   (a) Use the relativistic energy-momentum relation to show that the required relation is:
      \[ m_0c^2 = [(cp)^2 – K^2]/(2K) \, . \]
   (b) Show that this reduces to the expected result \( K = p^2/(2m_0) \) in the non-relativistic limit.
   (c) Find the rest mass, in units of the electron rest mass \( m_e \) (equal to 0.511 MeV/c\(^2\)), for a particle that has \( K = 55.0 \) MeV when \( p = 121 \) MeV/c.

**TR7.** (a) A particle of charge \( q \), rest mass \( m_0 \), speed \( v \), and Lorentz factor \( \gamma \) moves perpendicularly to a uniform magnetic field of magnitude \( B \). Show that the particle moves in a circle of radius \( r_L = \gamma m_0 v/(qB) \), the relativistic Larmor radius.
   (b) The proton synchrotron at Fermilab accelerates protons to a kinetic energy of 500 GeV, at which relativistic effects are very important. (The proton rest mass is 938.3 MeV/c\(^2\).) In particular, the period \( T \) of the proton’s circular orbit, of radius 750 m, in the synchrotron’s magnetic field increases as the proton accelerates. In a synchrotron (as distinct from a cyclotron) the magnetic field’s magnitude and oscillation frequency are varied to allow for this increase in \( T \). At the energy of 500 GeV, calculate:
      (i) the proton’s Lorentz factor,
      (ii) its speed parameter \( \beta \equiv v/c \), and
      (iii) the magnetic field strength.
from First-Year:

PHYS1102 - Special Relativity
Course Outline

Lecturer
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Recommended Reading
It is recommended that you read the sections from Young and Freedman given in brackets for each lecture.

Lecture Content

Lecture 1. Foundations of Special Relativity
events and inertial observers, the basic postulates of Special Relativity, simultaneity is not absolute, relativistic time dilation, relativistic length contraction. (Y & F sections 37.1, 37.2, 37.3, 37.4)

Lecture 2. Lorentz Transformations
form of the Lorentz transformations, deriving time dilation and length contraction from Lorentz transformations, relativistic addition of velocities parallel and perpendicular to the direction of relative motion. (Y & F section 37.5)

Lecture 3. Relativistic Energy and Momentum
relativistic mass, formulas for relativistic energy and momentum in terms of relativistic mass, Newtonian limit, the rest energy, the relativistic energy-momentum relation, particles which travel at the speed of light. (Y & F sections 37.7, 37.8)

Lecture 4. The Doppler Effect for Electromagnetic Radiation
derivation of formula for Doppler shift, application to Hubble’s law. (Y & F sections 37.6)

Lecture 5. The Spacetime Interval
relativistic invariants, the spacetime interval as a relativistic invariant, four-vectors, inner products of four-vectors, the four-vector of coordinate displacements between a pair of events.

Lecture 6. The Energy-Momentum Four-Vector
energy and momentum as components of a four-vector, Lorentz transformations of energy and momentum, the relativistic energy-momentum relation for a particle in terms of four-vectors, spacetime diagrams.

 see over for a First-Year formula sheet . . .
Length contraction: \( L = L_0 \sqrt{1 - v^2 / c^2} \), where \( L_0 \) is the length as measured by an observer at rest relative to the object, and \( L \) is the length as measured by an observer relative to whom the object is moving at speed \( v \). Length contraction only occurs in directions parallel to the direction of relative motion.

Time dilation: \( \Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}} \), where \( \Delta t_0 \) is the time interval between a pair of events measured by an observer relative to whom the two events take place at the same location in space, and \( \Delta t \) is the time interval between the same pair of events measured by an observer moving at speed \( v \) relative to the first observer.

Definition of gamma: \( \gamma = \frac{1}{\sqrt{1 - v^2 / c^2}} \)

For a free particle of rest mass \( m \), its energy is \( E = \gamma mc^2 \), and its momentum is \( \vec{p} = \gamma m \vec{v} \). The rest energy of the particle is \( E_0 = mc^2 \).

Relativistic energy-momentum relation for a particle of rest mass \( m \): \( E = \sqrt{\vec{p}^2 c^2 + m^2 c^4} \). For a particle which travels at the speed of light, \( E = |\vec{p}| c \).

Addition of velocities: \( O \) and \( O' \) are two observers in relative motion along their respective \( x \)-axes, and a body is in motion along the \( x \)-axis.

\[ u'_x = \frac{(u_x - v)}{(1 - u_x \gamma / c^2)} \]

where \( u_x \) is the velocity of the body relative to observer \( O \), and \( u'_x \) is the velocity of the body relative to observer \( O' \), and \( v \) is the velocity of observer \( O' \) relative to \( O \). The sign convention is that motion in the \( +x \) direction corresponds to a positive velocity.

Doppler shift for electromagnetic radiation: \( f = f_0 \sqrt{\frac{1 + u/c}{1 - u/c}} \)

Here, \( f_0 \) is the frequency measured by an observer at rest relative to the source, and \( f \) is the frequency measured by an observer toward whom the source is moving at speed \( u \). The sign convention is that \( u \) is positive for motion of the source toward the observer, and \( u \) is negative for motion of the source away from the observer.