CHAPTER 9

Threshold Requirements
Looking at Loss again

\[ \alpha = h \nu_c \frac{B}{c} (n_1 - n_2) = -\kappa \]

Gain/length = \( -\alpha \) (or \( \kappa \))

Loss/length = \( \alpha_1 \)

\[ I e^{-\left(\alpha_1 + \alpha\right)d} \]

All self-sustaining oscillators require gain to exceed losses

\[ IR_2 R_1 e^{-2(\alpha_1 + \alpha)d} \]

\[ IR_2 e^{-\left(\alpha_1 + \alpha\right)d} \]
Gain Threshold

\[
R_2 R_1 \ e^{2(\kappa - \alpha_1) d} \geq 1
\]

Eq 9.1

Rewrite this as:

\[
\kappa \geq \alpha_1 - \ln[R_2 R_1]/2d
\]

Atom stuff  Cavity stuff
Cavity Lifetime

(also called cold cavity lifetime or photon lifetime).

“the time the intensity decreases to 1/e of its starting value when there is no gain in the cavity”

In one round trip, intensity decreases by \( R_2 R_1 e^{2(\kappa - \alpha_1)d} \)

which takes \( 2d/c \) seconds (assuming cavity and gain material length are the same)

Assume intensity decrease can be described as \( e^{-t/t_c} \) where \( t_c \) is cold cavity lifetime

\[
t_c = \frac{2d}{c \left( 2\alpha_1 d - \ln [R_1R_2] \right)}
\]

Eq 9.3
Population Difference Threshold

\[ n_2 - n_1 \geq \frac{8\pi \nu r^2}{c^3 g(\nu r)} \frac{t_{sp}}{t_c} \]

Since achieving \( n_2 > n_1 \) is really hard, what should a smart person do?
Example

Ruby rod from before except let's use a 5cm long rod between two 90% reflectivity mirrors.

From Eq. 9.3 we get $t_c = 2.8 \text{ns}$, and therefore we find by substitution into Eq. 9.4 that:

$$n_2 - n_1 \geq 2 \times 10^{23} \text{ m}^{-3}$$

Typical Cr atom densities in ruby (about 1% replacement of Al) are around $1.6 \times 10^{25} \text{ m}^{-3}$.
Resonator Transfer Function in presence of gain

What happens after lots of round trips?
Go to Phet Simulation
Talking about Gain
Also, Talk about two level situation
Chapter 10

Pumping Processes and Rate Equations
Two Level Atom

Narrow Band Radiation at $\nu_r$ with density $u$, hit some atoms.

$$R_{\text{abs}} = n_1 c^3 A g(\nu_r) \frac{u(T)}{(8 \pi h \nu_r^3)} = n_1 W_{12}$$

$$R_{\text{em}} = n_2 c^3 A g(\nu_r) \frac{u(T)}{(8 \pi h \nu_r^3)} = n_2 W_{21}$$

N.B. $W_{12} = W_{21}$

$$\hat{n}_1 = W_{21} (n_2 - n_1) + A n_2$$
Solution for 2 level system

\[ \frac{n_2}{n_1} = \frac{W_{21}}{(W_{21} + A)} \]

Cannot possibly work!
Three Level Atom

Narrow Band Radiation at $\nu_p$ with density $u$.

$$\dot{n}_1 = W_{31}(n_3 - n_1) + A_{31}n_3 + A_{21}n_2 + W_{21}(n_2 - n_1)$$
Three Level Solution

\[
\frac{\Delta n}{N} = \frac{W_{31} - A_{21}}{W_{31} + A_{21}} \left(1 + \frac{1}{W_{12}} \frac{3W_{31} + 2A_{32}}{A_{32} (W_{31} + A_{21})}\right)
\]

Below threshold essentially no laser light: \( W_{12} = 0 \)

In this case pop difference, and hence gain, are indep. of \( W_{12} \)

\[\Rightarrow \text{ Intensity depends exponentially on distance} \]

Lets look at saturation in more detail in 4 level laser
**Example: pump power for Ruby**

At threshold:

\[
\frac{\Delta n}{N} \approx \frac{W_{31} - A_{21}}{W_{31} + A_{21}} \approx 0
\]

PD is nearly zero

No. of pumped atoms/sec/volume \(\sim W_{31} n_1\)

Power (per unit volume) will be:

\[
P = W_{31} n_1 h \nu_p = A_{21} N/2 h \nu_p
\]

For Ruby, \(A_{21} = 1/\tau = 1/(3\text{ms}), N \sim 1.6 \times 10^{19} \text{ cm}^{-3}\)

Pump wavelength \(\sim 480\text{nm}\) (frequency \(\sim 6 \times 10^{14} \text{ Hz}\))

\[
P = 1100 \text{ Wcm}^{-3}
\]

Electrical thresholds are about 16 times higher than this
\[ \dot{n}_1 = W_{41} (n_4 - n_1) + A_{41} n_4 + A_{21} n_2 \]

Smart: avoid using the ground state for one of the laser levels

If \( A_{21} > A_{43} \) then get PI for smallest pumping intensities
Four level solution

\[
\frac{\Delta n}{N} = \frac{W_{41}}{W_{41} + A_{32}} \left( 1 + \frac{2W_{41} + A_{21}}{A_{21} (W_{41} + A_{32})} \right)
\]

Small signal behaviour
(below and near threshold)

Viz. 3 level:
\[
\frac{\Delta n}{N} = \frac{W_{31} - A_{21}}{W_{31} + A_{21}} \left( 1 + \frac{3W_{31} + 2A_{32}}{A_{32} (W_{31} + A_{21})} \right)
\]
Nd:YAG example

NB. 2 errors in notes
p. 125 - should be N ~ 6 \times 10^{25} \text{ m}^{-3} (0.5\% \text{ doping})
p. 124 - should be Gain coeff/m on vertical axis of Fig 10.3

\lambda \sim 1064\text{nm}, \Delta \nu \sim 195 \text{GHz}, \tau \sim 230\mu\text{s}

N \sim 6 \times 10^{25} \text{ m}^{-3}, \quad n_0 \sim 1.82, \nu_p \sim 400 \text{THz}

R_1 \sim 100\%, \quad R_2 \sim 80\%, \quad l \sim 7\text{cm} \quad \alpha_1 = 0

\Delta n \sim 2.5 \times 10^{21} \text{m}^{-3}

W_{41} \sim 0.2 \text{s}^{-1}

P \sim W_{41} n_1 h \nu_p \sim W_{41} N h \nu_p \sim 3.2 \text{W/cm}^3
Gain dependence on output

Intensity (W/cm²)

W_{41} \sim 0.2 \text{ s}^{-1}

W_{41} \sim 0.4 \text{ s}^{-1}

W_{41} \sim 0.1 \text{ s}^{-1}

Threshold
No. of photons in mode

- Photon Number
- Pump Intensity (as ratio with threshold)

Diagram showing the relationship between photon number and pump intensity.
Power vs Current in LDs

- Hitachi 50 mW no extended cavity
- Hitachi 50 mW 1800 g/mm vis grating

Graph showing output power vs laser diode current (mA) and pump intensity (as ratio with threshold).
Radiation from a bunch of excited atoms

\[ \text{Intensity (a.u.)} \]

\[ \text{Time} \]

\[ \text{FWHM} = \frac{1}{2 \pi \tau} \]
Collisions

Intensity (a.u)

Time (fs)
Doppler Broadening

![Graph showing Doppler Broadening with intensity on the y-axis and frequency on the x-axis.](image)