Lecture 6

Introduction to Lasers and Masers & Optical Resonators at finite temperatures
“In some way they (the Martians) are able to generate an intense heat in a chamber of practically absolute nonconductivity. From this they project in a parallel beam against any object they choose, by means of a polished parabolic mirror of unknown composition...

However it is done, it is certain that a beam of heat is the essence of the matter. What is combustible flashes into flame at its touch, lead runs like water, it softens iron, crack and melts glass, and when it falls upon water, that explodes into steam.”
4kW laser cutting
1.5mm mild steel
Lasers/Masers

Lasers generate coherent radiation in the IR, visible or UV regions of the spectrum.

Masers operate in Microwave domain.

Light amplification by stimulated emission of radiation.
A Typical Laser

Three elements

R=100%
R=80%

Laser medium

pump

Lasers sizes range from 100nm (QD) through to many 100m
Laser Light

Spatial Coherence

Temporal Coherence (monochromatic)

Energy Density

Brightness

Broad Bandwidth
A metallic cavity…

\[ E_x(z) = A \sin (k_z z) \quad \text{where} \quad k_z = n_z \frac{\pi}{L_z} \]

also \( k_x = n_x \frac{\pi}{L_x} \) and \( k_y = n_y \frac{\pi}{L_y} \)

\[ k = \sqrt{k_x^2 + k_y^2 + k_z^2} \quad \text{and thus} \quad v_{nx,ny,nz} = c \frac{\sqrt{n_x^2 + n_y^2 + n_z^2}}{2L} \]
Masers

Let's consider mode frequencies in a 3cm cubical metallic cavity

\[ \nu_{110} = \nu_{101} = \nu_{011} = 7.1\text{GHz (4.2cm)} \]

\[ \nu_{111} = 8.7\text{GHz (3.5cm)} \]

\[ \nu_{210} = \nu_{201} = \nu_{012} = \nu_{021} = \nu_{102} = \nu_{120} = 11.2\text{ GHz (2.7cm)} \]

\[ \nu_{211} = \nu_{121} = \nu_{112} = 12.2\text{GHz (2.5cm)} \]

Masers were the original “lasers” and operate in the microwave domain. The active gain (for both lasers and masers) works over about 1-10% of the spectrum centered on some nominal frequency.
So perhaps only one mode can successfully interact with the gain in the system - and you will see only one output frequency and one shape.

What’s the situation with lasers? - will only be equivalent if we scaled the dimensions of the laser itself down by same factor of $10^5$. 
Laser Cavities (Dimension $\gg \lambda$)

We could write down every mode as we did down here…
But life is too short, so let's calculate it by using a differential approach - figure out the mode density

$\nu_{nx,ny,nz}$
1 mode/ unit volume

Only $n_{x,y,z} > 0$ make sense

Turns out there are 2 polarizations for each mode

$$\nu_{n_x,n_y,n_z} = \frac{c \sqrt{n_x^2 + n_y^2 + n_z^2}}{2L} = \frac{c}{2L} R$$

$$N' = \frac{2}{8} \frac{4}{3} \pi R^3 = \frac{8}{3} \pi \nu^3 \frac{L^3}{c^3}$$
\[ N' = \frac{8}{3} \pi \nu^3 \frac{L^3}{c^3} \]

i.e. total modes between 0 and \( \nu \) frequency

How many modes between two sphere’s \( d\nu \) apart?

\[ N'(\nu) = 8 \pi \nu^2 \frac{L^3}{c^3} d\nu \]

\[ N(\nu) = 8 \pi \frac{\nu^2}{c^3} d\nu \]
Mode density in 3D and 1D

\[ N(\nu) = 8 \pi \frac{\nu^2}{c^3} \, d\nu \]

In 1D cavity: \( \nu_q = \frac{q \, c}{2L} \)

Mode density: \( N(\nu)_{1D} = \frac{2L}{c} \, d\nu \)
### Example Cavities

<table>
<thead>
<tr>
<th></th>
<th>He-Ne Laser</th>
<th>Ruby Laser</th>
<th>GaAs DL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity L</td>
<td>0.2m</td>
<td>7.5cm</td>
<td>0.3mm</td>
</tr>
<tr>
<td>Cavity φ</td>
<td>3mm</td>
<td>1cm</td>
<td>1µ x 50µ</td>
</tr>
<tr>
<td>Gain Linewidth</td>
<td>1.5GHz</td>
<td>150GHz</td>
<td>8500GHz</td>
</tr>
<tr>
<td>(N(\nu)) (d\nu)</td>
<td>4 x 10^8</td>
<td>8 x 10^{11}</td>
<td>7 x 10^5</td>
</tr>
<tr>
<td>(N(\nu)_{1D} ) (d\nu)</td>
<td>2</td>
<td>10</td>
<td>17</td>
</tr>
</tbody>
</table>

In 1D cavity: \(\nu_q = q \frac{c}{2L}\)

Mode density: \(N(\nu)_{1D} = 2L c \, d\nu\)
CAVITIES WITH RADIATION
BLACK BODIES
EXPERIMENTAL BB CURVES

300, 500, 700, 900 K

\[ P_{\text{rad}} \sim \sigma T^4 \]

Explain Shape?
A cavity at temp. T: Classical Model

\[ U = (\text{number of degrees of freedom}) \times (\text{average energy per degree of freedom}) \]
Mode Energy in Equilibrium

Boltzmann says that a harmonic oscillator in equilibrium with thermal bath has an energy distribution:

\[ P(E) = \frac{1}{kT} \exp\left[-\frac{E}{kT}\right] \]

Mean energy of such an oscillator (mode)

\[ \bar{E} = \int E \, P(E) = kT \]

Combining mode density per volume and average energy per mode we get

\[ u(T, \nu) = 8\pi \nu^2 \frac{kT}{c^3} \]

(RJ formula):

\[ \int u(T, \nu) \, d\nu \to \infty \]
EXPERIMENT VS THEORY!!!

300, 500, 700, 900 K
A cavity at temp. $T$: Quantum Model
With special thanks to Mr. Planck and Mr. Einstein

Let's state that modes cannot have arbitrary energy but only a whole number of units of energy

- the unit is called the photon
- the number of photons ‘the occupation number’

Boltzmann is still right so we have:

\[
P(n) = (1 - \exp(-\frac{h \nu}{(kT)})) \exp[-n \frac{h \nu}{(kT)}]
\]

Mean energy:

\[
\overline{E} = \sum E_n P(n) = \frac{h \nu}{(\exp[h \nu / (kT)] - 1)}
\]

When \( h \nu < kT \) then this is same as RJ formula \( (e^x - 1 + x) \)
Rayleigh-Jeans vs Planck

$\frac{h\omega}{(kT)} \sim 0.2$

for $T \sim 300K$, $\omega \sim 10^{12}$
Rayleigh-Jeans vs Planck

300, 500, 700, 900 K

\[ \frac{h \omega}{kT} \sim 1.6 \]

for \( T \sim 300 \text{K} \), \( \omega \sim 10^{13} \)
The Planck Radiation Law

\[ u(T, \nu) = \frac{8 \pi \nu^2}{c^3} \frac{h \nu}{\text{Exp}[\frac{h \nu}{(kT)}] - 1} \text{ (J m}^{-3} \text{ Hz}^{-1}) \]

\[ h\omega/(kT) \approx 32 \]

for \( T \approx 300\text{K}, \omega \approx 2 \times 10^{14} \)

300, 500, 700, 900 K
Energy Density / Volume

From slide 13: \[ N(\nu) = \frac{8 \pi \nu^2}{c^3} \, d\nu \]

From slide 21: \[ E = \sum E_n \, P(n) = \frac{h \nu}{\exp[h \nu/(kT)] - 1} \]

Plank’s Radiation Law

\[ u(T,\nu) = \frac{8 \pi \nu^2}{c^3} \, \frac{h \nu}{\exp[h \nu/(kT)] - 1} \quad (J \, m^{-3} \, Hz^{-1}) \]